# A classical model for the negative *dc* conductivity of *ac*-driven 2D electrons near the cyclotron resonance

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A classical model for *dc* transport of two dimensional electrons in a perpendicular magnetic field and under strong irradiation is considered. We demonstrate that, near the cyclotron resonance condition, and for *linear* polarization of the *ac* field, a strong change of the diagonal component, $\sigma_d$, of the *dc* conductivity occurs in the presence of a *weak* nonparabolicity of the electron spectrum. Small change in the electron effective mass due to irradiation can lead to negative $\sigma_d$, while the Hall component of the *dc* conductivity remains practically unchanged. Within the model considered, the sign of $\sigma_d$ depends on the relative orientation of the *dc* and *ac* fields, the sign of the detuning of the *ac* frequency from the cyclotron resonance, and the sign of nonparabolic term in the energy spectrum. We also demonstrate that the known phenomenon of the nonparabolicity-induced hysteresis in the cyclotron absorption manifests itself in the *dc* transport by causing a hysteresis in the magnetic field dependence of $\sigma_d$.

1. *Introduction.* Recently reported observation [1,2] of a zero-resistance state, that emerges upon microwave irradiation of a high-mobility 2D electron gas in a weak magnetic field, was immediately followed by a number of theoretical papers [3–6], in which the origin of this state was discussed. The only microscopic calculation to date [4] indicates that, for strong enough radiation intensity, the diagonal component, $\sigma_d$, of the *dc* conductivity tensor changes sign from the dark value $\sigma_d > 0$ to $\sigma_d < 0$ within certain frequency intervals of the *ac* field, away from the cyclotron frequency and its harmonics. Negative local value of $\sigma_d$ results in the instability of the homogeneous current distribution. In Ref. [5] the scenario of how the instability might develop into the zero-resistance state was proposed.

In this situation it seems important to trace the emergence of negative $\sigma_d$ in an *ac*-driven system from the simplest possible model. Such a model is considered in the present paper. Obviously, $\sigma_d$ is sensitive to the illumination only if the Kohn theorem is violated. It is commonly assumed that the reason for this violation is a random impurity potential. Here we consider a model, in which the Kohn theorem is violated due to an intrinsic reason, namely, due to a weak nonparabolicity of the electron spectrum. More specifically, we adopt the following form of the dispersion relation for the conduction band electrons

$$\varepsilon(p) = \frac{p^2}{2m}\left[1 - \frac{p^2}{2mE_0}\right], \tag{1}$$

where $m$ is the effective mass, and $E_0$ is the energy of the order of the bandgap. The corresponding expression for the velocity reads

$$\boldsymbol{v} = \frac{\boldsymbol{p}}{m}\left[1 - \frac{p^2}{mE_0}\right]. \tag{2}$$

2. *dc conductivity.* As we will see below, negative $\sigma_d$ emerges when the *ac* field, $\boldsymbol{\mathcal{E}}\cos\omega t$, is linearly polarized. We choose the direction of polarization along the $x$-axis. We will also see that the

magnitude and the sign of $\sigma_d$ depend on the direction of the driving *dc* field, $\mathbf{E}$ (Fig. 1). Denote with $\theta$ the direction of $\mathbf{E}$ with respect to the $x$–axis. Then the classical equation of motion of an electron in a perpendicular magnetic field $\boldsymbol{B}$ and driven by *ac* and *dc* fields is

$$\frac{d\boldsymbol{p}}{dt} + \frac{\boldsymbol{p}}{\tau} - \frac{e}{c}\left[\boldsymbol{v} \times \boldsymbol{B}\right] = e\boldsymbol{E} + e\boldsymbol{\mathcal{E}}\cos\omega t, \tag{3}$$

where $\tau$ is the relaxation time. It is convenient to rewrite this equation introducing a complex variable $\mathcal{P} = p_x + ip_y$. Then it takes the form

$$\frac{d\mathcal{P}}{dt} + \frac{\mathcal{P}}{\tau} - i\omega_c\mathcal{P} + \frac{i\omega_c}{mE_0}\mathcal{P}|\mathcal{P}|^2 = eEe^{i\theta} + \frac{e\mathcal{E}}{2}\left(e^{i\omega t} + e^{-i\omega t}\right). \tag{4}$$

Here $\omega_c = |e|B/mc$ is the cyclotron frequency. We search for a solution of Eq. (4) in the form

$$\mathcal{P}(t) = \mathcal{P}_0 + \mathcal{P}_+\exp(i\omega t) + \mathcal{P}_-\exp(-i\omega t), \tag{5}$$

where $\mathcal{P}_0 \ll \mathcal{P}_+, \mathcal{P}_-$ is a small *dc* component proportional to $E$. Near the cyclotron resonance condition, $\omega \approx \omega_c$, we have $|\mathcal{P}_-| \ll |\mathcal{P}_+|$. Still we keep the nonresonant term, $\mathcal{P}_-$, to the lowest order, since the *ac* field affects $\sigma_d$ through this term. In other words, the effect of irradiation on $\sigma_d$ emerges beyond the rotating-wave approximation, adopted in Ref. [4]. Substituting Eq. (5) into Eq. (4), we obtain the following system of equations for $\mathcal{P}_+$, $\mathcal{P}_-$, and $\mathcal{P}_0$.

$$\left[i(\omega - \omega_c) + \frac{1}{\tau}\right]\mathcal{P}_+ + \frac{i\omega_c}{mE_0}\mathcal{P}_+|\mathcal{P}_+|^2 = \frac{e\mathcal{E}}{2}, \tag{6}$$

$$-\,i\left(\omega + \omega_c\right)\mathcal{P}_- = \frac{e\mathcal{E}}{2}, \tag{7}$$

$$\left[\frac{1}{\tau} - i\omega_c\left(1 - \frac{2|\mathcal{P}_+|^2}{mE_0}\right)\right]\mathcal{P}_0 + \left[\frac{2i\omega_c}{mE_0}\mathcal{P}_+\mathcal{P}_-\right]\mathcal{P}_0^* = eEe^{i\theta}. \tag{8}$$

The *ac*-induced term $\propto |\mathcal{P}_+|^2$ in the equation for $\mathcal{P}_0$ describes the change of the effective mass caused by irradiation, and has no effect on the *dc* transport. The effect comes from the term $\propto \mathcal{P}_+\mathcal{P}_-$, which describes the "rectification" of the Larmour motion due to nonparabolicity.

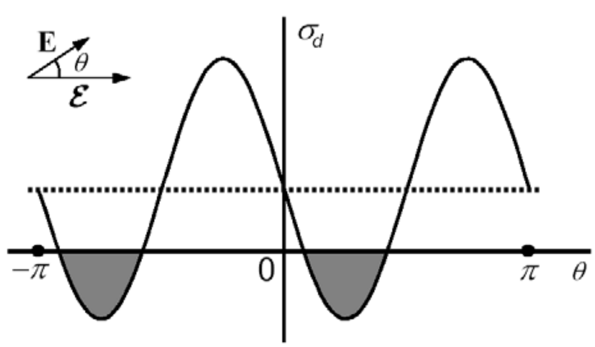


FIG. 1. The diagonal conductivity as a function of the angle $\theta$ between *ac* and *dc* electric fields, illustrated in the inset. The dashed line shows the dark conductivity. The solid curve is a plot of the diagonal conductivity under irradiation given by Eq. (14). It assumes negative values within the angular intervals shown with gray.

From Eq. (7) we obtain $\mathcal{P}_- = ie\mathcal{E}/4\omega_c$. The solution of Eq. (6) can be formally presented in the form

$$\mathcal{P}_+ = \frac{e\mathcal{E}\tau}{2(1+i\Omega\tau)}, \tag{9}$$

where

$$\Omega = \omega - \omega_c + \frac{\omega_c|\mathcal{P}_+|^2}{mE_0} \tag{10}$$

is the deviation of the microwave frequency from the *ac*–shifted cyclotron frequency. Substituting $\mathcal{P}_+$, $\mathcal{P}_-$ into Eq. (8), we find the longitudinal, $p_\parallel$, and transverse, $p_\perp$, with respect to the applied *dc* field, components of the drift momentum

$$p_\parallel = Re\left\{\mathcal{P}_0 e^{-i\theta}\right\} = \frac{eE}{\omega_c^2\tau}\left\{1 - \left[\frac{(e\mathcal{E}\tau)^2}{8mE_0}\right]\frac{\Omega\tau\sin 2\theta - \cos 2\theta}{1+(\Omega\tau)^2}\right\}, \tag{11}$$

$$p_\perp = Im\left\{\mathcal{P}_0 e^{-i\theta}\right\} = \frac{eE}{\omega_c}\left\{1 - \left[\frac{(e\mathcal{E})^2\tau}{8mE_0\omega_c}\right]\frac{\Omega\tau\cos 2\theta + \sin 2\theta}{1+(\Omega\tau)^2}\right\}. \tag{12}$$

The expressions for the diagonal, $\sigma_d$, and transverse, $\sigma_t$, conductivities immediately follow from Eqs. (11), (12). It is convenient to rewrite these expressions using the dimensionless parameter $\delta m/m$, which is the relative correction to the effective mass due to irradiation

$$\frac{\delta m}{m} = \frac{|\mathcal{P}_+|^2}{mE_0} = \frac{(e\mathcal{E}\tau)^2}{4mE_0\left[1+(\Omega\tau)^2\right]}. \tag{13}$$

The above consideration is valid when $\delta m/m \ll 1$. Then we obtain

$$\sigma_d = \frac{ne^2}{m\omega_c}\left[\frac{1}{\omega_c\tau} - \left(\frac{\delta m}{2m}\right)\frac{\Omega\tau\sin 2\theta - \cos 2\theta}{\omega_c\tau}\right], \tag{14}$$

$$\sigma_t = \frac{ne^2}{m\omega_c}\left[1 - \left(\frac{\delta m}{2m}\right)\frac{\Omega\tau\cos 2\theta + \sin 2\theta}{\omega_c\tau}\right], \tag{15}$$

where $n$ is the electron concentration. We see that the *ac*-induced correction to the transverse conductivity is small, since we assumed that $|\Omega| \ll \omega_c$ and $\delta m/m \ll 1$. The latter condition allows us to use the expansion of $\varepsilon(p)$ given by Eq. (1), i.e., to neglect the higher–order, in $\varepsilon/E_0$, terms in the kinetic energy. Our prime observation, however, is that with $\delta m/m \ll 1$, the diagonal conductivity, given by Eq. (14), becomes *negative* for large enough detuning from the cyclotron resonance. The corresponding condition for negative $\sigma_d$ can be presented as

$$\frac{|\Omega|}{\omega_c} > \frac{2}{\omega_c\tau}\left(\frac{m}{\delta m}\right). \tag{16}$$

In the clean limit, $\omega_c\tau \gg 1$, this condition is compatible with $|\Omega| \ll \omega_c$ and $\delta m/m \ll 1$. The ratio $\delta m/m$ increases with the intensity of the *ac* field. Therefore, in order to meet the condition (16), microwave irradiation should exceed a critical value. For the intensities above this critical value the diagonal conductivity assumes negative values within certain intervals of relative orientation, $\theta$, as illustrated in Fig. 1.

It is instructive to analyze the above expressions for diagonal and transverse conductivities in the limit $\tau \to \infty$, where they can be simplified to

$$\sigma_d = -\left(\frac{ne^2}{m\omega_c}\right)\left[\frac{(e\mathcal{E})^2}{8mE_0\Omega\omega_c}\right]\sin 2\theta, \tag{17}$$

$$\sigma_t = \frac{ne^2}{m\omega_c}\left[1 - \frac{(e\mathcal{E})^2}{8mE_0\Omega\omega_c}\cos 2\theta\right]. \tag{18}$$

Remarkably, the relaxation time, $\tau$, drops out not only from $\sigma_t$, but also from the diagonal conductivity. This means that the the momentum relaxation, necessary for dissipative transport, is provided by scattering from the microwave field, coupled to the translational motion via the nonparabolic term in the dispersion relation.

As it is seen from Eq. (17), the dissipation is the *odd* function of the detuning, $\Omega$, from the cyclotron resonance. Within the interval $0 < \theta < \pi/4$ it is negative for $\omega > \omega_c$ and positive for $\omega < \omega_c$.

3. *Bistability.* In general, $\sigma_d$ in Eq. (17) is not simply proportional to the intensity of the *ac* field. This is because the effective detuning, $\Omega$, also depends on $\mathcal{E}$, as follows from Eq. (10). Below we analyze the $\Omega(\mathcal{E})$ dependence. For this purpose, we rewrite Eq. (9) in the form

$$z \equiv \frac{|\mathcal{P}_+|^2}{mE_0} = \frac{|e\mathcal{E}|^2}{4mE_0\omega_c^2(\delta+z)^2}, \qquad \delta \equiv \frac{\omega-\omega_c}{\omega_c}. \tag{19}$$

Upon introducing new dimensionless variables

$$\eta = \frac{|e\mathcal{E}|^2}{4mE_0\omega_c^2}, \qquad s = -z/\delta, \tag{20}$$

this equation takes the form

$$s(1-s)^2 = -\frac{\eta}{\delta^3}, \tag{21}$$

Diagonal conductivity can be expressed through a solution of this equation as follows

$$\sigma_d = -\frac{1}{2}\sigma_t\delta^2 s(s-1)\sin 2\theta, \tag{22}$$

where $\sigma_t = ne^2/m\omega_c$ is the transverse conductivity. Remarkably, Eq. (21) yields three solutions for $\delta < 0$ and $\eta/|\delta|^3 < 4/27$. Two of these solutions are stable. As a result, the nonparabolicity Eq. (1) in the dispersion law of the conduction band leads to the hysteresis in the cyclotron absorption. This fact was first pointed out in Ref. [7]. Later it was noticed in Ref. [8] that the hysteresis in the cyclotron absorption is actually possible for a *free* electron due to relativistic correction to its velocity. The role of $\tau$ in Ref. [8] is played by the radiative friction. This prediction was confirmed experimentally in Ref. [9].

The aspect of nonlinear cyclotron resonance [7,8], which is interesting to us, is how the bistability manifests itself in the diagonal conductivity, when polarization of the *ac* field is linear. Two stable solutions of Eq. (21) can be obtained analytically in the limit $\eta \ll |\delta|^3$

$$s_1 = \frac{\eta}{|\delta|^3}, \qquad s_2 = 1 + \left(\frac{\eta}{|\delta|^3}\right)^{1/2}. \tag{23}$$

It is seen from Eq. (22) that they result in two values of diagonal conductivity

$$\sigma_{d1} = \sigma_t \frac{\eta}{2|\delta|} \sin 2\theta, \quad \sigma_{d2} = -\frac{1}{2}\sigma_t \sqrt{\eta|\delta|} \sin 2\theta. \tag{24}$$

This result is obtained in the limit of infinite $\tau$. Analysis with finite $\tau$ indicates that as long as the condition $\delta m/m \ll 1$ is met, the phenomenon of negative diagonal conductivity vanishes for the second solution, while $\sigma_{d1}$ remains unaffected. On the other hand, kinetic energies of these states are

$$\varepsilon_1 = \frac{\eta}{2\delta^2} E_0, \quad \varepsilon_2 = \frac{1}{2}|\delta| E_0. \tag{25}$$

Note, that the condition $\eta \ll |\delta|^3$ insures that $\varepsilon_1 \ll \varepsilon_2$. Our findings can be thus summarized as follows: negative $\sigma_d$ corresponds to the lower–energy state, while in the higher–energy state the diagonal conductivity remains positive. Hence, within the model considered in the present paper, the phenomena resulting from negative diagonal conductivity should vanish at high enough temperature due to activation to the state with positive $\sigma_d$.

4. *Concluding remarks.* We would like to emphasize that the effect of irradiation on *dc* transport emerges within our model only for linear (more precisely, not circular) polarization of the *ac* field. As a result, the *ac*–induced contribution depends on the relative orientation of the *dc* field and *ac*–polarization.

As the intensity of irradiation increases, the diagonal conductivity first becomes zero at certain orientation, $\theta$, which depends on the detuning from the cyclotron resonance. Upon further growth of the microwave intensity, the interval of $\theta$ within which $\sigma_d$ is negative, becomes increasingly wider. We also emphasize that, within our simple model, a significant change in the dissipative conductivity with irradiation occurs only for the *ac* frequency near the cyclotron resonance. Experimentally observed oscillations of $\sigma_d$ in the vicinity of the harmonics of the cyclotron resonance are not captured by our model.

Uncertainty in the experimental parameters does not allow a detailed comparison of our model to the experiments [1,2]. We can only roughly estimate the crucial parameter $(e\mathcal{E}\tau)^2/mE_0$, which should me compared to $(\omega_c\tau)^{-1}$ in order to achieve the negative value of one of the diagonal components of the condictivity tensor. Using the value $100\mu W$ for the microwave power from Ref. [2], and sample area $10^{-2}cm^2$, we find for the microwave electric field $\mathcal{E} \sim 1V/cm$. Note, that the *ac* field was linearly polarized in both Refs. [1] and [2]. For the experimental mobility $\mu = 2.5 \cdot 10^7 cm^2/V sec$ and for nonparabolicity parameter $E_0 = 1eV$, we obtain for the dimensionless combination $(e\mathcal{E}\tau)^2/mE_0$ the value of 0.03, which is quite reasonable, since $(\omega_c\tau)$ is $\sim 50$ in Ref. [2]. The estimate turns out reasonable because the smallness of nonparabolicity in GaAs is "compensated" by the long scattering time in very high mobility samples studied in Refs. [1,2].

Finally, we would like to illustrate with a numerical example how the hysteresis in the the cyclotron absorption line [7,8] translates into the hysteresis in the magnetic field dependence of the *dc* conductivity. In Fig. 2a the dimensionless kinetic energy $z = |\mathcal{P}_+|^2/mE_0$ of the *ac* driven electron is plotted vs. the dimensionless detuning from the cyclotron resonance condition. The dependence is calculated for the dimensionless intensity $\eta = 2 \cdot 10^{-3}$ of the *ac* field, defined by Eq. (20), and for $\omega_c\tau = 30$. In Fig. 2b the corresponding dependence of the *dc* conductivity (in the units of the "dark" value $\sigma_d^{(0)} = ne^2/m\omega_c^2\tau$) is is shown for the orientation $\theta = \pi/4$. It is seen that the jump between the low-energy and the high-energy states of the Larmour rotation is accompanied by the change of the sign of the *dc* conductivity.

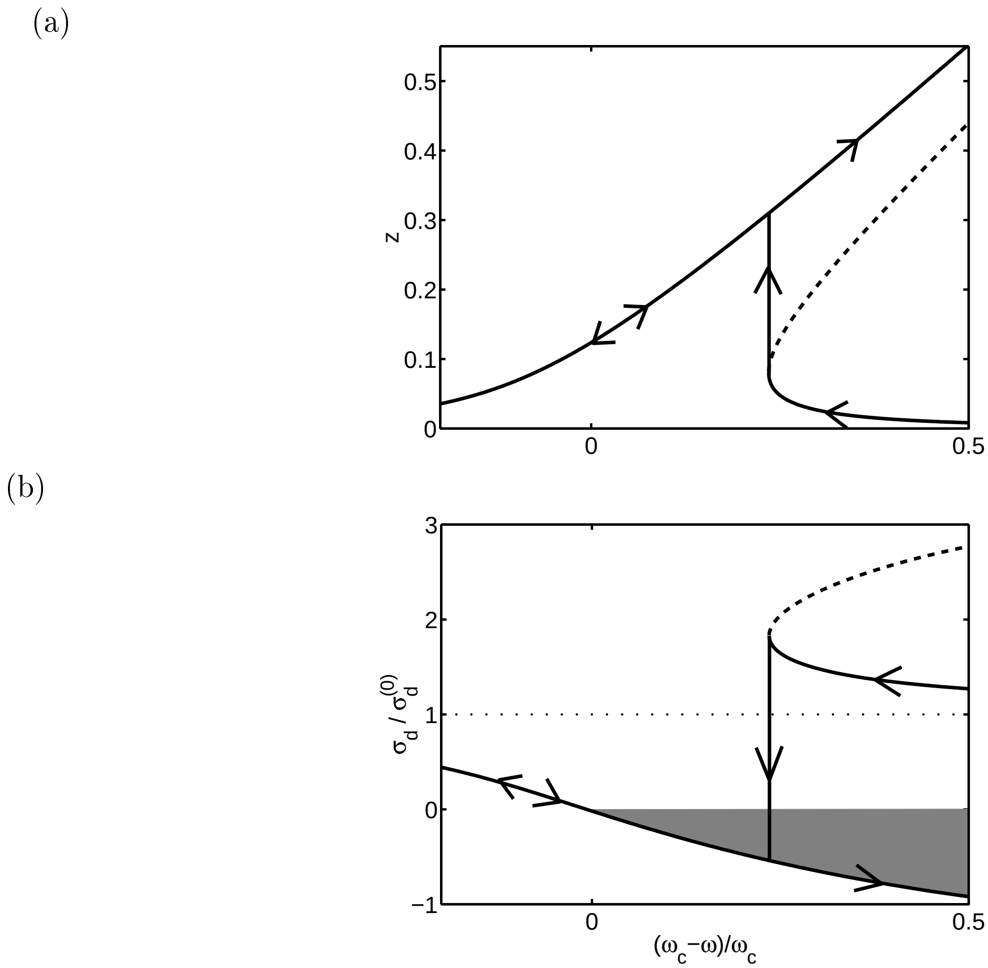


FIG. 2. Hysteresis in the dimensionless kinetic energy (a) and the dimensionless *dc* conductivity (b) is illustrated for $\omega_c\tau = 30$, dimensionless intensity $\eta = 2 \cdot 10^{-3}$, and orientation $\theta = \pi/4$, of the *ac* field.

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